RESONANT FREQUENCY OF DIELECTRIC RESONATORS IN INHOMOGENEOUS MEDIA*

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Abstract

An iterative analytical method is presented for the computation of resonant frequencies of high dielectric constant cylindrical resonators in inhomogeneous media. Theoretical calculations of barium tetratitanate dielectric discs above alumina substrates of a microstrip line agree with measurements of the resonant frequency to within 0.2 percent.

Introduction

Interest in the utilization of high dielectric constant resonators has been revived recently because of the availability of low-loss temperature stable materials.[1,2] Several approximate methods have been presented for determining the resonant frequency when such resonators are placed between two conducting planes perpendicular to the cylindrical resonator's axis.[3-5] In practice, when a resonator is used in a microwave circuit employing a microstrip transmission medium, the microstrip substrate, as well as other boundaries, significantly alters the resonant frequency predicted from the idealized conditions usually assumed. This paper describes an iterative technique for the calculation of the dielectric resonator's resonant frequency (assuming a $TE_{10\delta}$ mode) which considers the substrate material, the ground planes, other dielectric supports, and the metallic boundaries surrounding the resonator.

The method imposes inhomogeneous boundary conditions on the fields in the axial direction to compute the δ of the $TE_{10\delta}$ mode. Then, the wave impedances at the resonator's radial surface are matched inside and outside the dielectric material. This process gives a new corrected resonant frequency, which can again be used to compute a new δ of the $TE_{10\delta}$ mode thereby continuing the iteration. The method converges rapidly to the correct resonant frequency.

Analysis

Figure 1 shows the configuration under analysis, consisting of a cylindrical resonator (region 3) with high ε_r positioned between three layers of (different) dielectrics (regions 1, 2, and 4). The field components for the $TE_{10\delta}$ mode in the ith region (i = 1 to 5) are the following:

$$H_z^{(i)} = C_0(k_i r)\; g_i(z) \qquad (1)$$

$$H_r^{(i)} = \frac{1}{k_i}\; C_0'(k_i r)\; g_i'(z) \qquad (2)$$

$$E_\phi^{(i)} = \frac{j\omega\mu_0}{k_i}\; C_0'(k_i r)\; g_i(z) \qquad (3)$$

$$H_\phi^{(i)} = E_r^{(i)} = E_z^{(i)} = 0 \qquad (4)$$

with

$$C_0(x) = J_0(x) \quad \text{for } r \le R_1$$

$$C_0(x) = K_0(x) - \frac{K_0[x(R_2/r)]\; I_0(x)}{I_0[x(R_2/r)]} \quad \text{for } R_1 \le r \le R_2$$

where J_0, K_0, and I_0 are the appropriate Bessel functions, and

$$g_i(z) = A_i \sinh \xi_i z + B_i \cosh \xi_i z \quad \text{for } i = 1,2,4$$

$$g_i(z) = A_i \sin \xi_i z + B_i \cos \xi_i z \quad \text{for } i = 3,5$$

The radial and axial wave numbers (k_i and ξ_i) are related by the wave equation as

$$k_i^2 = \omega^2\mu_0\varepsilon_i \pm \xi_i^2 \qquad (5)$$

The minus sign in equation (5) holds for i = 3 and 5. For the region in which $R_1 \le r \le R_2$, the fields are assumed to be below cut-off in the radial direction, so that

$$(jk_5)^2 = \xi_5^2 - \omega^2\mu_0\varepsilon_5$$

is a real number.

Of the ten unknown constants (A_i's and B_i's) eight are related by the eight boundary conditions for the tangential fields at the boundary surfaces in the z direction. The existence of nonzero solutions for the above linear system implies that

$$\frac{t_3t_4 - 1}{\xi_3t_4/\xi_4 + t_3} - \frac{\xi_4/\xi_3 - pt_3}{p + \xi_4/\xi_3 t_3} = 0 \qquad (6)$$

where

$$t_3 = tg\xi_3h_2/2$$

$$t_4 = tgh\xi_4d_2$$

$$p = \frac{(\xi_2/\xi_1 tgh\xi_1h_1 + tgh\xi_2d_1)}{(1 + \xi_2/\xi_1 tgh\xi_2d_1)}$$

The relationship between the ξ_i follows from

$$k_1 = k_2 = k_3 = k_4$$

Since

$$\xi_3 = \delta\pi/h_2$$

where δ defines the mode,

$$\xi_i^2 = \omega^2\mu_0(\varepsilon_3 - \varepsilon_i) - (\delta\pi/h_2)^2 \quad i = 1,2,4$$

$$\xi_5 = \delta\pi/h_2 \qquad (7)$$

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Equation (6) has, therefore, two unknowns, δ and ω. Another equation results from the continuity of the fields at the dielectric disk edge, which can be written in terms of the wall admittance matching

$$\overrightarrow{Y} + \overleftarrow{Y} = 0 \qquad (8)$$

where

$$\overrightarrow{Y} = \left. \frac{H_z^{(5)}}{E_\phi^{(5)}} \right|_{r=R_1} , \qquad \overleftarrow{Y} = - \left. \frac{H_z^{(3)}}{E_\phi^{(3)}} \right|_{r=R_1}$$

Equation (8) together with equations (1) through (3) yields

$$k_3 \frac{J_0(k_3R_1)}{J_0'(k_3R_1)} + jk_5 \frac{C_0(jk_5R_1)}{C_0'(jk_5R_1)} = 0 \qquad (9)$$

Results

Equations (6) and (9) were solved iteratively. The first step is searching for a value of δ that satisfies equation (6) at a given ω and then computing ω to satisfy equation (9) for this value of δ and so on. No more than six iterations are necessary to arrive at a stationary result. The starting frequency was computed from the work of Yee.[7]

Figure 1 shows the theoretical and experimental results obtained with a $BaTi_4O_3$ dielectric pill ($\varepsilon_3 = 36.8$) enclosed in a cylindrical cavity with a moving top. The pill was held by a cylindrical foam with $\varepsilon_5 \simeq 1.0$ over an alumina substrate with $\varepsilon_1 = 10.0$. For this case, ε_2 equals ε_4 equals 1.0. The coupling was performed by 50Ω microstrip lines etched in the alumina substrate, and offset from the axis of the disc.

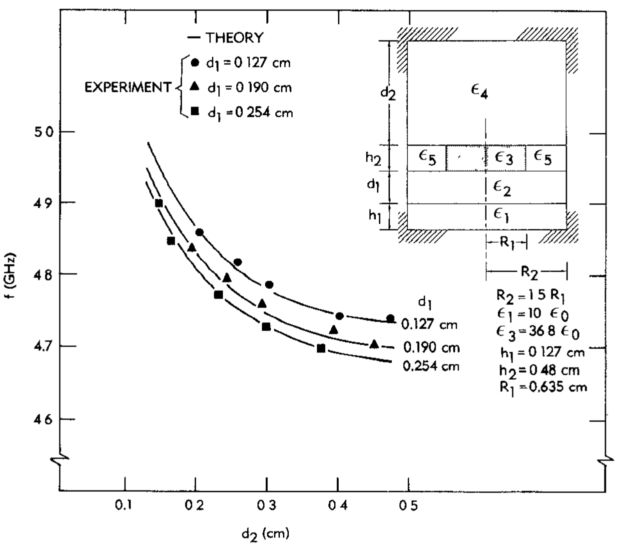

Figure 1. Experiment and Theoretical Results

Conclusions

An accurate iterative solution to the problem of finding the resonant frequency of a high dielectric cylindrical resonator for inhomogeneous media has been presented. Results of the method agree closely with measurements.

References

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